

# Calc Q2 Test 3 Review Key

1)  $f(x) = 4x^3 - x^4$

$f'(x) = 12x^2 - 4x^3$

$0 = 12x^2 - 4x^3$

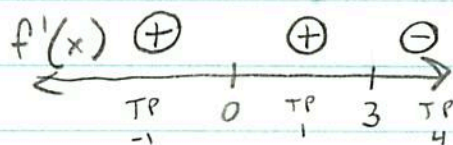
$0 = 4x^2(3-x)$

$x=0 \quad | \quad x=3$

$f''(x) = 24x - 12x^2$

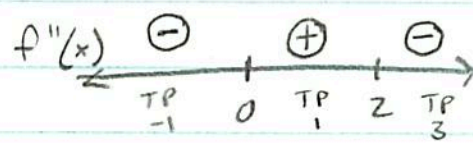
$0 = 12x(2-x)$

$x=0 \quad | \quad x=2$



a)  $(-\infty, 0) \cup (0, 3)$

b)  $(3, \infty)$



c)  $(0, 2)$

d)  $(-\infty, 0) \cup (2, \infty)$

e) Inflection Pts

$(0, 0)$  and  $(2, 16)$

$f(0) = 4(0)^3 - (0)^4$

$f(2) = 4(2)^3 - 2^4$

2)  $f(x) = \frac{2x}{x-2}$

$f'(x) = \frac{(x-2)(2) - (2x)(1)}{(x-2)^2}$

$f''(x) = \frac{(x-2)^2(0) - (-4)(2(x-2)(1))}{(x-2)^4}$

$f'(x) = \frac{2x-4-2x}{(x-2)^2}$

$f''(x) = \frac{0 + 8(x-2)}{(x-2)^4}$

$f'(x) = \frac{-4}{(x-2)^2}$

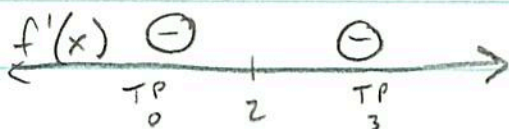
$f''(x) = \frac{8(x-2)}{(x-2)^4} = \frac{8}{(x-2)^3}$

$f'(x) = 0$   
Never

$f'(x) = \text{und.}$   
 $x-2=0$   
 $x=2$

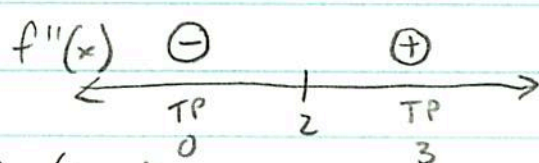
$f''(x) = 0$   
Never

$f''(x) = \text{und.}$   
 $x-2=0$   
 $x=2$



a) Never

b)  $(-\infty, 2) \cup (2, \infty)$



c)  $(2, \infty)$

d)  $(-\infty, 2)$

(No inflection pt)

e) Inflection pt at  $x=2$

But function DNE there

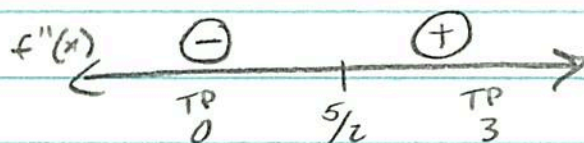
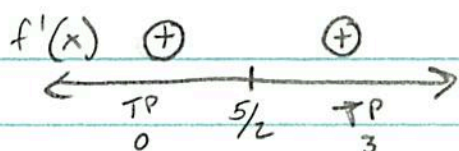
$$3) f(x) = (2x-5)^3 \quad f'(x) = 3(2x-5)^2(2) \quad f''(x) = 12(2x-5)(2)$$

$$f'(x) = 6(2x-5)^2 \quad f''(x) = 24(2x-5)$$

$$0 = 6(2x-5)^2 \quad 0 = 24(2x-5)$$

$$0 = 2x-5 \quad | \quad x = \frac{5}{2}$$

$$\frac{5}{2} = x$$



a)  $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

b) Never

c)  $(\frac{5}{2}, \infty)$

d)  $(-\infty, \frac{5}{2})$

e) Inflection P+ @  $x = \frac{5}{2}$

$$f(\frac{5}{2}) = (2(\frac{5}{2}) - 5)^3$$

$$f(\frac{5}{2}) = 0$$

$$(\frac{5}{2}, 0)$$

$$4) f(x) = \frac{x^2}{x^2-4} \quad f'(x) = \frac{(x^2-4)(2x) - (x^2)(2x)}{(x^2-4)^2} \quad f''(x) = \frac{(x^2-4)^2(-8) - (-8x)(2)(x^2-4)(2x)}{(x^2-4)^4}$$

$$f'(x) = \frac{2x^3 - 8x - 2x^3}{(x^2-4)^2}$$

$$f''(x) = \frac{-8(x^2-4)^2 + 32x^2(x^2-4)}{(x^2-4)^4}$$

$$f'(x) = \frac{-8x}{(x^2-4)^2}$$

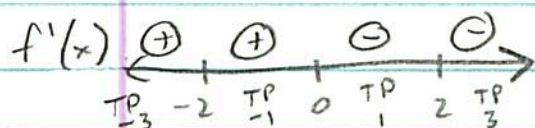
$$f''(x) = \frac{-8(x^2-4) + 32x^2}{(x^2-4)^3}$$

$$f'(x) = 0 \quad f'(x) = \text{und}$$

$$-8x = 0 \quad x^2 - 4 = 0$$

$$x = 0 \quad x = \pm 2$$

$$f''(x) = \frac{-8x^2 + 32 + 32x^2}{(x^2-4)^3} = \frac{24x^2 + 32}{(x^2-4)^3}$$



$$f''(x) = 0$$

$$24x^2 + 32 = 0$$

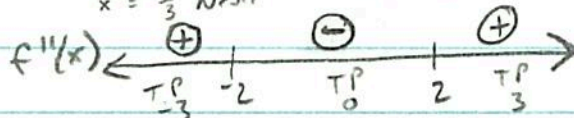
$$8(3x^2 + 4) = 0$$

$$x^2 = -\frac{4}{3} \text{ Never}$$

$$f''(x) = \text{und}$$

$$x^2 - 4 = 0$$

$$x = \pm 2$$



a)  $(-\infty, -2) \cup (-2, 0)$

b)  $(0, 2) \cup (2, \infty)$

c)  $(-\infty, -2) \cup (2, \infty)$  d)  $(-2, 2)$  e) None



5)  $f(x)$  is increasing when  $f'(x)$  is positive (above the x-axis)  
 $(-1, 3) \cup (5, \infty)$

6)  $f(x)$  is decreasing when  $f'(x)$  is negative (below the x-axis)  
 $(-\infty, -1) \cup (3, 5)$

b)  $f(x)$  is concave up when  $f'(x)$  is increasing  
 $(-\infty, \frac{1}{2}) \cup (4, \infty)$

$f(x)$  is concave down when  $f'(x)$  is decreasing  
 $(\frac{1}{2}, 4)$

c)  $f(x)$  has an inflection pt when  $f'(x)$  has a max or min  
Inflection pt when  $x = \frac{1}{2}$  and  $x = 4$

7) I  $\rightarrow$  C

II  $\rightarrow$  B

III  $\rightarrow$  A

8) a)  $f(x) = 2x^3 - 9x^2 + 12x - 5$   $[0, 2]$   $f(0) = 2(0)^3 - 9(0)^2 + 12(0) - 5 = -5$  Abs Min  
 $f'(x) = 6x^2 - 18x + 12$   $f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 5 = 0$  Abs Max  
 $0 = 6(x^2 - 3x + 2)$   $f(2) = 2(2)^3 - 9(2)^2 + 12(2) - 5 = -1$   
 $(x-2)(x-1)$   
 $x=2$   $x=1$

b)  $f(x) = 2x^3 - 15x^2 + 24x + 2$   $[0, 2]$   $f(0) = 2(0)^3 - 15(0)^2 + 24(0) + 2 = 2$  Abs Min  
 $f'(x) = 6x^2 - 30x + 24$   $f(1) = 2(1)^3 - 15(1)^2 + 24(1) + 2 = 13$  Abs Max  
 $6(x^2 - 5x + 4)$   $f(2) = 2(2)^3 - 15(2)^2 + 24(2) + 2 = 6$   
 $(x-4)(x-1)$   
 $x=4$   $x=1$

Not in  
interval

9) a)  $f'(-2) < 0$

b)  $f'(-1) > 0$

c)  $f'(0) = 0$

d)  $f(1.5) > 0$

e)  $f'(-3) > f'(-1)$       Tangent line @  $f'(-3)$  is steeper than @  $f'(-1)$

f)  $f(1) > f'(1)$        $f(1) = 0$

g)  $f''(1) > f'(1)$        $f(x)$  is concave up at  $x=1$   
therefore  $f''(x)$  is positive

h)  $f'(1) < f'(0)$